

## GAS–CONDENSATE FLOW IN THE VICINITY OF A HYDRAULIC FRACTURE

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*The problem of gas–condensate flow in the vicinity of a production well with a hydraulic fracture is considered. In the matrix, the flow is assumed to be three-dimensional, and at the fracture, it is assumed to be two-dimensional. It is shown that, for steady-state flow, the problem is split into a physicochemical problem (of phase transitions) and a filtration problem (of determining the pressure field). Numerical solutions are constructed for a rectangular fracture with finite and infinite conductivities.*

**Key words:** *gas–condensate mixture, fluid fracture, filtration, steady-state flow.*

**Introduction.** In gas–condensate reservoirs, a situation often occurs when the reservoir hydrocarbon mixture is in a steady single-phase (gaseous) state under the initial thermobaric conditions. During exploitation, the reservoir pressure declines, the mixture becomes thermodynamically unstable, and a liquid phase (condensate) appears (this phenomenon is called retrograde condensation [1, 2]). From the point of view of reservoir performance, condensate formation is an unfavorable factor. First, transition of higher hydrocarbons to a liquid phase, as a rule, results in their irreversible losses in the reservoir; second, the occurrence of the liquid phase leads to deterioration of the rock transport properties and, hence, a reduction in well productivity. To optimize the exploitation of gas–condensate reservoirs, it is of interest to analyze the exact solutions of the problem of filtration of a two-phase multicomponent mixture.

In the present work, we study the filtration of a gas–condensate mixture in the vicinity of a hydraulic fracture. Equations describing the flow of a gas–condensate mixture in a porous medium containing a permeable fracture are formulated. The properties of steady-state flows are studied. Because the problem of steady-state filtration of a gas–condensate mixture in the one-dimensional and two-dimensional cases is integrated in quadratures [3–6], the properties of the exact solutions can be used to interpret stationary studies of gas–condensate mixtures and predict well productivity [7]. The method of solving one-dimensional and two-dimensional problems [3–6] is extended to the three-dimensional case. Some numerical results for filtration problems in the vicinity of the hydraulic fracture are given.

**1. Basic Equations.** We assume that an  $M$ -component gas–condensate mixture fills a spatial domain  $D$  with a piecewise smooth boundary  $\partial D$  in a porous bed; the indices  $a$ ,  $b$ , and  $c$  corresponding to the ordinal numbers of the space coordinates  $x^a$  (which may not be Cartesian) take values 1, 2, and 3, and the indices  $i$ ,  $j$ , and  $k$  corresponding to the mixture component numbers take values  $1, \dots, M$ . The number of moles of the  $i$ th component of the mixture in unit volume will be denoted by  $n_i$ , and the mass of the mole of the  $i$ th component by  $m_i$ . The permeable fracture described by a two-dimensional surface  $\Gamma$  propagates through the domain  $D$ . The surface  $\Gamma$  is assumed to be smooth, i.e., possible effects of the fractal dimension [8] are ignored. The indices  $\alpha$ ,  $\beta$ , and  $\gamma$  corresponding to the ordinal numbers of the curvilinear coordinates  $\xi^\alpha$  on the surface  $\Gamma$  take values 1 and 2. In the adopted notation, the surface  $\Gamma$  is described by the equations  $x^a = X^a(\xi^\alpha)$ . Summation is performed over repeated indices that correspond to the coordinates or numbers of the components. We use the following

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designations:  $g_{ab} = g_{ab}(x^c)$  are the covariant components of the metric tensor in space;  $g_{\alpha\beta*} = g_{\alpha\beta*}(\xi^\gamma)$  are the covariant components of the metric tensor on the surface  $\Gamma$ ,  $g = \det(g_{ab})$ ,  $g_* = \det(g_{\alpha\beta*})$ ,  $\partial_a = \partial/\partial x^a$ ,  $\partial_{\alpha*} = \partial/\partial \xi^\alpha$ ,  $Z_{,i} = \partial Z/\partial n_i$ , and  $\nabla_a$  and  $\nabla_{\alpha*}$  are the covariant derivatives in space and on the surface  $\Gamma$  (the Levi-Civita connection for the metrics  $g_{ab}$  and  $g_{\alpha\beta*}$ , respectively [9]). We note that the metrics  $g_{ab}$  ( $g_{\alpha\beta*}$ ) can be used to raise and lower the indices of the tensor fields in the domain  $D$  (on the surface  $\Gamma$ ) [10]. Because we consider only isothermal flows, the temperature dependence of the parameters is ignored.

We assume that  $n_{ig} = n_{ig}(t, x^a)$  and  $n_{ic} = n_{ic}(t, x^a)$  are the densities of the gas and condensate components in the matrix (measured in moles per unit volume),  $n_{ig*} = n_{ig*}(t, \xi^\alpha)$  and  $n_{ic*} = n_{ic*}(t, \xi^\alpha)$  are the densities of the gas and condensate components in the fracture,  $s_g = s_g(t, x^a)$  and  $s_c = s_c(t, x^a)$  are the gas and condensate saturations in the matrix ( $s_g + s_c = 1$ ), and  $s_{g*} = s_{g*}(t, \xi^\alpha)$  and  $s_{c*} = s_{c*}(t, \xi^\alpha)$  are the gas and condensate saturations in the fracture ( $s_{g*} + s_{c*} = 1$ ).

For the mixture considered, the free energy per unit volume  $f = f(n_i)$ , which depends on the molar densities of the components  $n_i$ , is determined. In application to particular problems, the function  $f = f(n_i)$  is calculated using semiempirical equations of state [1, 2]. Knowing the free energy and the Gibbs–Duhem relation

$$dp = n_i d\chi_i, \quad (1)$$

it is possible to calculate the chemical potential of a mixture component  $\chi_i = f_{,i}$  and the hydrostatic pressure  $p = n_i \chi_i - f$ .

The densities of the components are used to calculate the chemical potentials of the components in the gas and condensate  $\chi_{ig} = \chi_i(n_{jg})$  and  $\chi_{ic} = \chi_i(n_{jc})$  and the phase pressure  $p_g = p(n_{ig})$ ,  $p_c = p(n_{ic})$  in the matrix. The chemical potentials of the gas and condensate components [ $\chi_{ig*} = \chi_i(n_{jg*})$  and  $\chi_{ic*} = \chi_i(n_{jc*})$ ] and the phase pressures [ $p_{g*} = p(n_{ig*})$  and  $p_{c*} = p(n_{ic*})$ ] in the fracture are calculated similarly. We assume that, in the absence of capillary forces in the matrix and fracture, the conditions of local thermodynamic equilibrium of phases are satisfied:

$$\chi_{ig} = \chi_{ic}, \quad p_g = p_c; \quad (2)$$

$$\chi_{ig*} = \chi_{ic*}, \quad p_{g*} = p_{c*}. \quad (3)$$

In addition, we assume that the porous medium is homogenous and isotropic, the porosity coefficient  $m$  does not depend on pressure, the fracture opening is specified as a smooth field on the surface  $\Gamma$ :  $h_* = h_*(\xi^\alpha)$ , and the medium filling the fracture is characterized by a constant porosity coefficient  $m_*$ . Then, the total free energy of the mixture is written as

$$F = m \int_D \left( s_g f(n_{ig}) + s_c f(n_{ic}) + (s_g n_{ig} + s_c n_{ic}) m_i \varphi \right) g dx^1 dx^2 dx^3 \\ + m_* \int_\Gamma \left( s_{g*} f(n_{ig*}) + s_{c*} f(n_{ic*}) + (s_{g*} n_{ig*} + s_{c*} n_{ic*}) m_i \varphi \right) h_* g_* d\xi^1 d\xi^2, \quad (4)$$

where  $\varphi = \varphi(x^a)$  is the gravity potential.

The filtration of the gas–condensate mixture should satisfy the conditions of local preservation of the components in the matrix

$$m \partial_t (s_g n_{ig} + s_c n_{ic}) + \nabla_a I_i^a = 0 \quad (5)$$

and in the fracture

$$m_* h_* \partial_t (s_{g*} n_{ig*} + s_{c*} n_{ic*}) + \nabla_{\alpha*} I_{i*}^{\alpha} + [I_i^{\alpha} l_a] = 0. \quad (6)$$

Here  $I_i^a$  and  $I_{i*}^{\alpha}$  are the fluxes of the  $i$ th component in the matrix and fracture and  $l_a$  is the unit normal to the surface  $\Gamma$ ; square brackets denote a discontinuity of the quantity. The discontinuity is calculated as the difference of the values of this parameter in the direction of the vector  $l_a$  and the value of this parameter in the opposite direction.

We assume that the boundary  $\partial\Gamma$  of the surface  $\Gamma$  is a piecewise smooth curve. Let  $k^\alpha$  be a unit inward normal to  $\partial\Gamma$  in the geometry of the surface  $\Gamma$ ,  $ds$  a measure on  $\partial\Gamma$ ,  $\gamma_1 = \partial\Gamma \cap \partial D$  the intersection of the fracture

surface with the boundary of the spatial domain  $D$ , and  $\gamma_2 = \partial\Gamma - \gamma_1$  the part of the fracture boundary inside the domain  $D$ . We also assume that the internal fluxes  $I_{i*}^\alpha$  vanish on the curve  $\gamma_2$ .

Using the dynamic equations (5) and (6), the phase equilibrium conditions (2) and (3), and the adopted assumptions, we calculate the time derivative of the total free energy of the mixture (4):

$$\frac{dF}{dt} = \Sigma_1 + \Sigma_2, \quad (7)$$

$$\Sigma_1 = \int_{\partial D} K^a I_i^a (\varkappa_i + m_i \varphi) dA + \int_{\gamma_1} k_\alpha I_{i*}^\alpha (\varkappa_{i*} + m_i \varphi) ds;$$

$$\Sigma_2 = \int_D I_i^a \partial_a (\varkappa_i + m_i \varphi) g dx^1 dx^2 dx^3 + \int_\Gamma \left( I_{i*}^\alpha \partial_{\alpha*} (\varkappa_i + m_i \varphi) + (\varkappa_i - \varkappa_{i*}) [I_i^a l_a] \right) g_* d\xi^1 d\xi^2 \quad (8)$$

( $K^a$  is the unit inward normal to the surface  $\partial D$ ). In (7), the term  $\Sigma_1$  describes the change in the free energy due to the flow through the boundary of the domain, and the term  $\Sigma_2$  describes the change in the free energy due to the processes inside the domain. For isothermal processes, an analog of the well-known condition of nonnegative entropy production is the inequality

$$\Sigma_2 \leq 0. \quad (9)$$

For the fluxes, we assume that the components are transferred by the flow of the phases in the pores:

$$I_i^a = n_{ig} u_g^a + n_{ic} u_c^a; \quad (10)$$

$$I_{i*}^\alpha = h_* (n_{ig*} u_{g*}^\alpha + n_{ic*} u_{c*}^\alpha). \quad (11)$$

Here  $u_g^a$  and  $u_c^a$  are the filtration velocities of the gas and condensate in the matrix and  $u_{g*}^\alpha$  and  $u_{c*}^\alpha$  are the filtration velocities of the gas and condensate in the fracture. In addition, we assume that the chemical potentials in the fracture and matrix are equal:

$$\varkappa_{i*} = \varkappa_i \Big|_\Gamma. \quad (12)$$

From relation (12) it follows that the densities of the components in the phases and the pressures for the mixture in the matrix and fracture coincide:

$$n_{ig*} = n_{ig} \Big|_\Gamma, \quad n_{ic*} = n_{ic} \Big|_\Gamma, \quad p_* = p \Big|_\Gamma. \quad (13)$$

Generally, however, the saturations in the fracture and matrix can differ:

$$s_{g*} \neq s_g \Big|_\Gamma.$$

Using relations (1), (10), (11), and (13), we transform expression (8) to obtain

$$\begin{aligned} \Sigma_2 &= \int_D \left( u_g^a (\partial_a p + \rho_g \partial_a \varphi) + u_c^a (\partial_a p + \rho_c \partial_a \varphi) \right) g dx^1 dx^2 dx^3 \\ &+ \int_\Gamma \left( u_{g*}^\alpha (\partial_{\alpha*} p + \rho_g \partial_{\alpha*} \varphi) + u_{c*}^\alpha (\partial_{\alpha*} p + \rho_c \partial_{\alpha*} \varphi) \right) g_* d\xi^1 d\xi^2, \end{aligned} \quad (14)$$

where  $\rho_g = m_i n_{ig}$  and  $\rho_c = m_i n_{ic}$  are the mass densities of the gas and condensate, respectively.

From (14) it follows that in order for inequality (9) to be valid, it is sufficient that the phases in the matrix and fracture obey the Darcy law:

$$u_g^a = -k f_g \mu_g^{-1} g^{ab} (\partial_b p + \rho_g \partial_b \varphi); \quad (15)$$

$$u_c^a = -k f_c \mu_c^{-1} g^{ab} (\partial_b p + \rho_c \partial_b \varphi); \quad (16)$$

$$u_{g*}^\alpha = -k_* f_{g*} \mu_g^{-1} g^{\alpha\beta} (\partial_{\beta*} p + \rho_g \partial_{\beta*} \varphi); \quad (17)$$

$$u_{c*}^\alpha = -k_* f_{c*} \mu_c^{-1} g^{\alpha\beta} (\partial_{\beta*} p + \rho_c \partial_{\beta*} \varphi). \quad (18)$$

Here  $k$  and  $k_*$  are the absolute permeabilities of the matrix and fracture,  $f_g$  and  $f_c$  are the relative permeabilities of the gas and condensate in the matrix, and  $f_{g*}$  and  $f_{c*}$  are the relative permeabilities in the fracture. It is assumed that the relative permeabilities are specified as functions of the condensate saturation. According to the assumption that the porous bed is homogeneous, the absolute permeability of the matrix  $k$  does not depend on the coordinates. However, the absolute permeability of the medium filling the fracture is generally a function on the surface  $\Gamma$ :  $k_* = k_*(\xi^\alpha)$ .

Relations (2), (10), (11), (13), and (15)–(18) close the dynamic problem (5), (6). Different physically meaningful formulations of the problems are possible which differ in the geometry of the domain  $D$  and the surface  $\Gamma$  and boundary and initial conditions.

It should be noted that relations (12) and (15)–(18) are not a unique possible set of relations that close the problem and are compatible with condition (9). For example, there are complicated filtration models with filtration velocities dependent nonlinearly on the pressure gradient. In the model proposed in the present section, the determining relations have the simplest analytical form and the model corresponds to a large number of laboratory and full-scale observations.

**2. Analytical Properties of Steady-State Solutions.** We examine steady-state solutions of the problem which correspond to steady-state filtration flows of the gas–condensate mixture. In this case, the conservation equations for the components (5) and (6) reduce to the conditions

$$\nabla_a I_i^a = 0; \quad (19)$$

$$\nabla_{\alpha*} I_{i*}^\alpha + [I_i^a l_a] = 0. \quad (20)$$

We assume that the surface  $\Gamma$  is inside the domain  $D$  and that the two-dimensional boundary  $\partial D$  of the domain  $D$  consists of piecewise smooth surfaces  $S_1$  and  $S_2$ . On the surface  $S_1$ , we specify a constant pressure  $p_r$  that corresponds to reservoir pressure, and on the surface  $S_2$ , the flux condition through this surface. Because the surface  $\Gamma$  describes the hydraulic fracture, it should geometrically be related to the production well. In the formulation considered, the trajectory of the well bore is described by a curve  $L$  on the surface  $\Gamma$ . On the curve  $L$ , a constant pressure  $p_w$  corresponding to the bottom hole pressure is assumed. Gravity forces are neglected.

To analyze Eq. (19), it is convenient to choose a coordinate system in space such that the conditions  $x^1 = p$ ,  $g_{1\alpha} = 0$  are satisfied and that the coordinates  $x^2, x^3$  at the fracture coincide with the internal coordinates of the surface  $\Gamma$ :  $x^2 = \xi^1$ ,  $x^3 = \xi^2$ . This coordinate system can be defined by setting the coordinates  $x^2$  and  $x^3$  to constant along the streamlines (i.e along the pressure gradient field lines). In the calculations, however, it is necessary to take into account that the mapping of the domain  $D$  onto the corresponding domain in the coordinates  $p$  and  $\xi^\alpha$  is two-sheeted because, for the same values of the parameters  $\xi^1$  and  $\xi^2$ , the streamline can approach a fracture from two sides.

In the chosen coordinate system, the fracture is described by an equation of the form  $p = p_*(\xi^\alpha)$ , the boundary of the domain  $\partial D$  by the equation  $p = p_r$ , the well  $L$  by the equation  $p = p_w$ , and the metric form in the space, by the definition, has the form

$$ds^2 = P^2 dp^2 + \sigma_{\alpha\beta} d\xi^\alpha d\xi^\beta. \quad (21)$$

The metric form on the fracture  $\Gamma$  is calculated by the formula

$$g_{\alpha\beta*} = P^2 \partial_{\alpha*} p_* \partial_{\beta*} p_* + \sigma_{\alpha\beta}.$$

Using the metrics (21) and system (19), in the chosen coordinate system, we write

$$\frac{\partial}{\partial p} \left( \frac{\sigma^{1/2}}{P} k(B_g c_{ig} + B_c c_{ic}) \right) = 0,$$

$$B_g = f_g \mu_g^{-1} n_g, \quad B_c = f_c \mu_c^{-1} n_c, \quad n_g = \sum_{i=1}^M n_{ig}, \quad n_c = \sum_{i=1}^M n_{ic}, \quad (22)$$

$$c_{ig} = n_{ig}/n_g, \quad c_{ic} = n_{ic}/n_c, \quad \sigma = \det(\sigma_{\alpha\beta}).$$

System (22) has  $M$  first integrals

$$\sigma^{1/2}k(B_g c_{ig} + B_c c_{ic})/P = q_i(\xi^\alpha). \quad (23)$$

The right sides of these equations represent the component fluxes that enter the domain  $D$  through the boundary of the domain  $\partial D$ . In the case where the reservoir pressure  $p_r$  is higher than or equal to the saturation pressure  $p_D$ , the condensate is absent on the boundary  $\partial D$  ( $B_c = 0$ ) and the right sides are proportional to the gas concentration  $c_{i0}$ :

$$q_i = c_{i0}q(\xi^\alpha). \quad (24)$$

If the reservoir pressure is lower than the saturation pressure  $p_D$  and, therefore, on the boundary  $\partial D$  there is condensate, relations (24) are assumed to be valid but the set of concentrations  $c_{i0}$  is interpreted as the composition of the moving part of the reservoir mixture.

The total flux to the well is calculated as an integral of the flux density over the boundary  $\partial D$ :

$$Q = \int_{\partial D} P^{-1}k(B_g + B_c)\sigma^{1/2} d\xi^1 d\xi^2 = \int_{\partial D} q(\xi^\alpha) d\xi^1 d\xi^2.$$

Using relation (24), we transform the set of integrals (23) to obtain

$$\begin{aligned} A_g c_{ig} + A_c c_{ic} &= c_{i0}, \\ A_g &= \sigma^{1/2} P^{-1} q^{-1} k B_g, \quad A_c = \sigma^{1/2} P^{-1} q^{-1} k B_c. \end{aligned} \quad (25)$$

Relations (25) are balance relations for the decomposition of the mixture with concentration  $c_{i0}$  into a gas and a condensate with concentrations  $c_{ig}$  and  $c_{ic}$ , respectively. Thus, the following representation holds:

$$A_g = 1 - W, \quad A_c = W. \quad (26)$$

Here, the function  $W = W(p)$ , representing the mole fraction of the condensate in the mixture with mean concentration  $c_{i0}$ , can be determined, independently of the filtration problem, from experiment or calculations using a semiempirical equation of state [1, 2]. Similarly, all characteristics of the gas and condensate  $c_{ig}$ ,  $c_{ic}$ ,  $n_g$ ,  $n_c$ ,  $\mu_g$ , and  $\mu_c$  can be found as functions of the pressure  $p$ .

Relations (26) imply the following equality containing no metric coefficients:

$$f_c/f_g = W\mu_c n_g (1 - W)^{-1} \mu_g^{-1} n_c^{-1}. \quad (27)$$

The right side of this equality contains a function of the pressure  $p$ , and the left side contains a function of the condensate saturation in the matrix  $s_c$ . Thus, relation (27) can be treated as an equation that describes the dependence of the saturation  $s_c$  on the pressure  $p$ .

From the aforesaid, it follows that, before solving the proper filtration problem, which results in the spatial distribution of the parameters of the mixture, one can obtain the dependences  $B_g = B_g(p)$  and  $B_c = B_c(p)$ . Denoting  $\Phi = \Phi(p) = B_g(p) + B_c(p)$ , from (19) it is easy to obtain the elliptic equation for the matrix pressure:

$$0 = k^{-1} \sum_{i=1}^M \nabla_a I_i^a = g^{ab} \nabla_a (\Phi(p) \nabla_b p). \quad (28)$$

Using the results obtained for the component fluxes in the matrix, we transform system (20) as follows:

$$\begin{aligned} g_*^{\alpha\beta} k^{-1} \nabla_{\alpha*} (k_* h_* (B_{g*} c_{ig} + B_{c*} c_{ic}) \nabla_{\beta*} p) &= -\Phi c_{i0} [l^a \partial_a p], \\ B_{g*} &= f_{g*} \mu_g^{-1} n_g, \quad B_{c*} = f_{c*} \mu_c^{-1} n_c. \end{aligned} \quad (29)$$

The right side of system (29) is proportional to the constant vector  $c_{i0}$ ; therefore, we can seek a solution of the problem in which the expression under the derivative side on the left of the system, which depends on the chemical component number, is also proportional to the vector  $c_{i0}$ . In this case, similarly to (25), we obtain the balance relations

$$\begin{aligned} A_{g*} c_{ig} + A_{c*} c_{ic} &= c_{i0}, \\ A_{g*} &= \Phi_*^{-1} B_{g*}, \quad A_{c*} = \Phi_*^{-1} B_{c*}, \quad \Phi_* = B_{g*} + B_{c*}. \end{aligned}$$

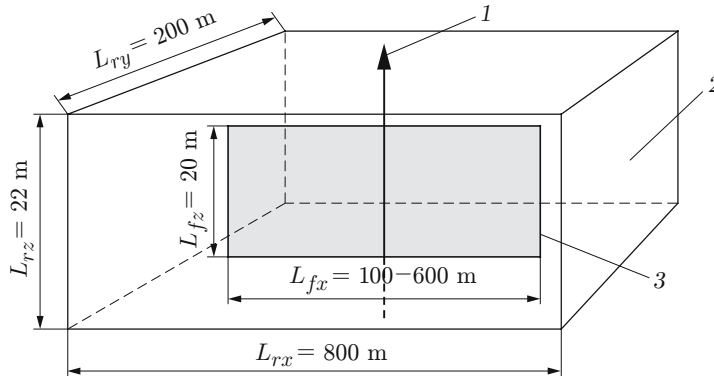


Fig. 1. Geometry of the calculation domain and fracture: well (1), computational domain (2), and fracture (3).

Similarly to (26), the relations  $A_{g*} = 1 - W$  and  $A_{c*} = W$  are valid. Therefore, the dependence of the condensate saturation in the fracture  $s_{c*}$  on the pressure  $p$  can be determined from the analog of Eq. (27):

$$f_{c*}/f_{g*} = W \mu_c n_g (1 - W)^{-1} \mu_g^{-1} n_c^{-1}.$$

Thus, the quantity  $\Phi_*$  is determined as a function of the pressure  $p$ . Summing Eqs. (29) by the component numbers, we obtain a differential relation for the pressure  $p$  on the surface  $\Gamma$  that is the internal boundary conditions for problem (28):

$$g_*^{\alpha\beta} k^{-1} \nabla_{\alpha*} (k_* h_* \Phi_* \nabla_{\beta*} p) + \Phi [l^a \partial_a p] = 0. \quad (30)$$

According to the aforesaid, in addition to condition (30), the pressure  $p$  is subjected to the boundary conditions

$$p|_{S_1} = p_r, \quad \lambda^a \partial_a p|_{S_2} = 0; \quad (31)$$

$$p|_L = p_w, \quad (32)$$

where  $\lambda^a$  is the normal to the surface  $S_2$ .

In the particular case where the fracture conductivity is high enough ( $k_*/k \rightarrow +\infty$ ), the problem reduces to Eq. (28), boundary conditions (31), and the additional boundary condition

$$p|_{\Gamma} = p_w,$$

which replaces conditions (30) and (32). Thus, the problem of filtration of a gas–condensate mixture in the vicinity of a hydraulic fracture reduces to a nonlinear elliptic equation with boundary conditions containing a nonlinear elliptical operator on the fracture [see (30)]. Generally, this problem can be solved only using numerical methods. Nevertheless, the resulting pressure problem is much simpler than the initial problem (19), (20), which contains unknown phase concentrations and saturations. It should be noted that Eqs. (28) and (30) are written in invariant form which does not depend on the employed coordinate systems in space and on the surface  $\Gamma$ , although these equations were derived using a particular coordinate system [see (21)].

**3. Some Numerical Solutions of the Problem.** In the formulation set forth in Sec. 2, the filtration problem was solved numerically for a mixture similar in composition to the composition of the mixture produced at object No. 2 of the Karachaganak oil–gas–condensate field (Republic of Kazakhstan). The mixture has the following molar composition:  $c_{N_2} = 0.0103$ ,  $c_{CO_2} = 0.0462$ ,  $c_{H_2S} = 0.0432$ ,  $c_{CH_4} = 0.6269$ ,  $c_{C_2H_6} = 0.0822$ ,  $c_{C_3H_8} = 0.0308$ ,  $c_{nC_4H_{10}} = 0.0062$ ,  $c_{iC_4H_{10}} = 0.0103$ ,  $c_{C_5} = 0.0285$ ,  $c_{C_6} = 0.0149$ , and  $c_{C_{7+}} = 0.1005$ .

The adopted reservoir pressure  $p_r = 53.5$  MPa is close to the saturation pressure of 53.0 MPa, and the bottom hole pressure is  $p_w = 41.5$  MPa. The thermodynamic characteristics and phase transitions were calculated by the Peng–Robinson equation of state [1, 2]. The viscosities of the gas and condensate were set constant:  $\mu_g = 2.3 \cdot 10^{-5}$  Pa · sec and  $\mu_c = 4.9 \cdot 10^{-4}$  Pa · sec.

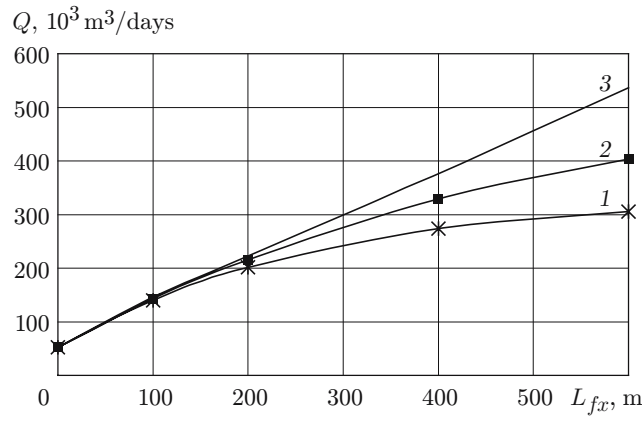


Fig. 2. Gas flow rate versus fracture length  $L_{fx}$ : curves 1 and 2 refer to  $k_*h_* = 400$  and  $1000 \text{ D} \cdot \text{mm}$ , respectively, and curve 3 refers to  $k_*h_* = \infty$ .

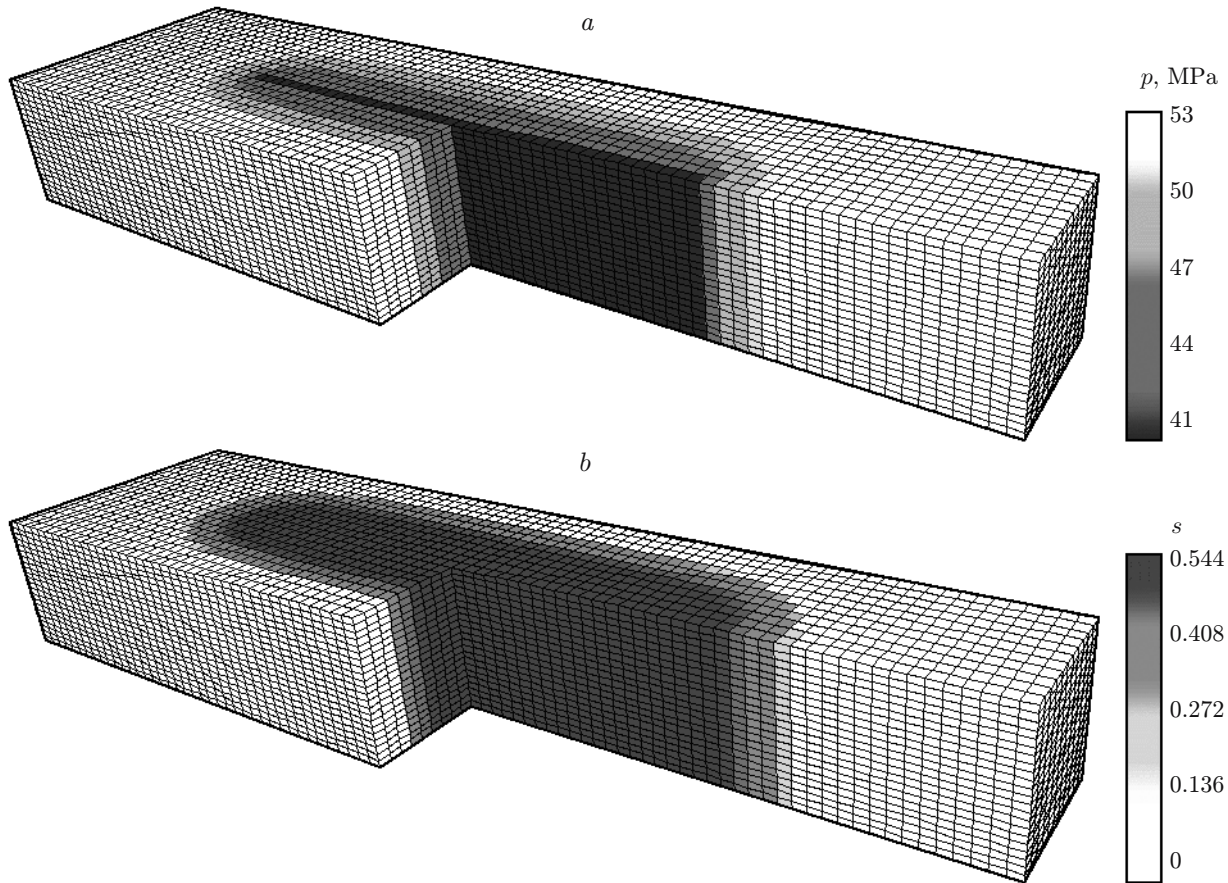


Fig. 3. Pressure (a) and condensate saturation (b) distributions in a fracture of infinite conductivity at  $L_{fx} = 400 \text{ m}$ .

The numerical modeling was performed for a rectangular plane fracture of varied lengths for fixed dimensions of the computational domain (Fig. 1). The upper and lower faces of the computational domain were assumed to be impervious, and the pressure on the lateral faces was equal to the reservoir pressure. The results [3] were obtained for a perfect fracture (of infinite conductivity  $k_*h_*$ ) and an imperfect fracture (of finite conductivity  $k_*h_*$ ). For the imperfect fracture, we used two conductivity values 400 D · mm or 1000 D · mm, and the permeability of the matrix was set equal to 1 mD (here and below 1 D = 1 Darcy is the permeation achieved by the passage of 1 ml of fluid of 1-cP viscosity flowing in 1 sec). The employed finite-difference grid contained a three-dimensional computational domain with dimensions of 800 × 200 × 22 m, which was divided into 80 × 20 × 22 cells, and a two-dimensional fracture with dimensions of (100–600) × 20 m, which was divided into (10–60) × 20 cells.

In the calculation, the following pressure dependences of the relative permeabilities were used:

$$f_g = (s_g - s_{g1})^a / (1 - s_{g1})^a, \quad f_c = (s_c - s_{c1})^b / (1 - s_{c1})^b.$$

Here  $s_{g1}$  and  $s_{c1}$  are the threshold mobilities for the gas and condensate, respectively. Various relative permeabilities were used:

- in the matrix,  $a = 2$ ,  $b = 3$ ,  $s_{g1} = 0.08$ , and  $s_{c1} = 0.12$ ;
- on the fracture,  $a = 2$ ,  $b = 2$ ,  $s_{g1} = 0$ , and  $s_{c1} = 0$ .

The calculation results are presented in Fig. 2. It is evident that the curves of the well flow rate versus fracture length are monotonically increasing. For the imperfect fracture, the curves enter an asymptote and, beginning from a certain length, the fracture growth does not lead to a considerable increase in the flow rate. Thus, for the specified conductivity, there may be a choice of the optimal fracture length. For the perfect fracture, the dependence of the flow rate on the fracture length is almost linear.

Figure 3 shows the pressure and condensate saturation distribution. It is evident that a condensate roll forms near the fracture with a considerable pressure gradient in this region.

**Conclusions.** The model proposed here for the filtration of a gas–condensate mixture in the vicinity of a hydraulic fracture allows effective predictions of the well productivity and distributions of mixture parameters in the matrix and fracture, which can be used in designing fluid fracturing.

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